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Numerical Simulation of Fast Transient Phenomena in Fluid-Structure Systems

A Short Course by F. Casadei

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The Joint Research Centre **(JRC)**

7 Institutes in 5 Member States:

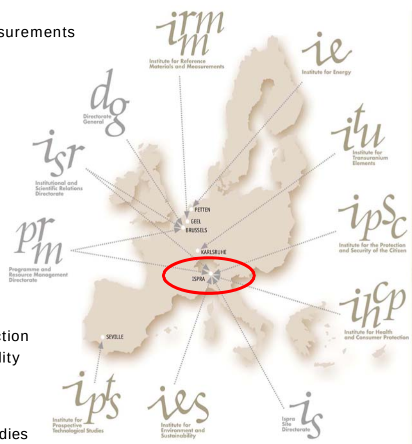
IRMM – Geel, Belgium
- Institute for Reference Materials and Measurements

IE – Petten, The Netherlands
- Institute for Energy

ITU – Karlsruhe, Germany
- Institute for Transuranium elements

IPSC - IHCP - IES – Ispra, Italy
- **Institute for the Protection and Security of the Citizen**
- Institute for Health and Consumer Protection
- Institute for Environment and Sustainability

IPTS – Seville, Spain
-Institute for Prospective Technological Studies



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The European Laboratory for Structural Assessment **(ELSA)**

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ELSA

Facilities:

Reaction Wall ("**Slow**" Dynamics)

Large Hopkinson Bar ("**Fast**" Dynamics)

“Slow” dynamics: earthquake …

Kobe (Japan) earthquake. January 17, 1995 (6,300 fatalities)

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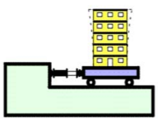
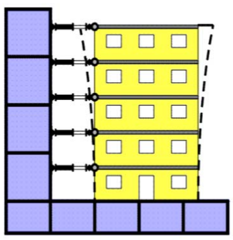
Earthquake testing of structures:

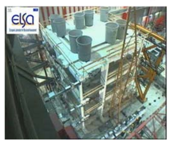
two complementary methods

Shaking Table



Reaction Wall





- Dynamic test
  (real time)
- Small-scale models

- Pseudo-dynamic test
  (expanded time scale)
- Full-scale models

(ELSA)

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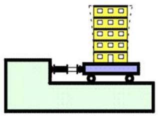
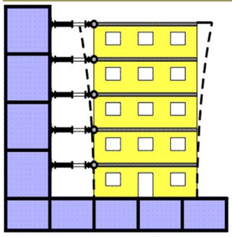
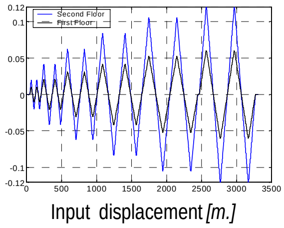
Imposed motion

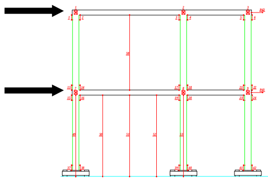
Shaking Table
Base motion

Reaction Wall
Floors motion

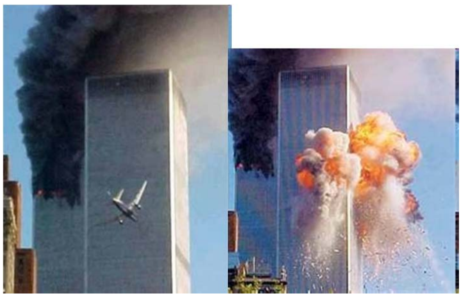






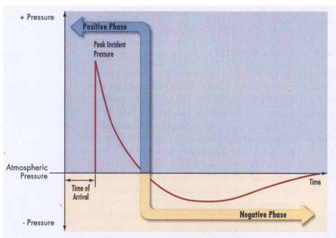


"Fast" dynamics: impact, explosion …



Twin towers. September 11, 2001

(2,967 fatalities)



Progressive Collapse: a “slow” phenomenon often initiated by a “fast” transient

Twin towers.

Structure collapse

Murrah Building, Oklahoma City. April 19, 1995 (168 fatalities)

Aladdin Hotel, Las Vegas. April 27, 1998 (controlled demolition)

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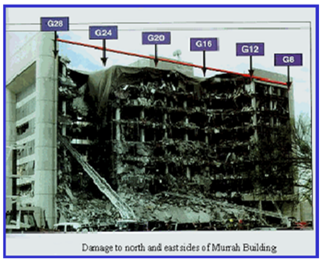
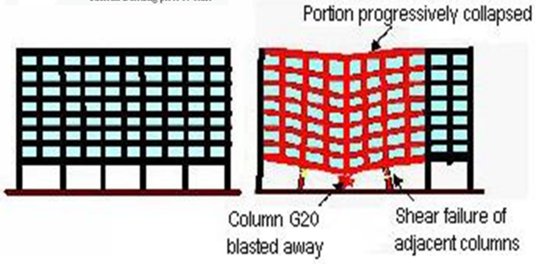
Example of Progressive Collapse due to terrorist attack





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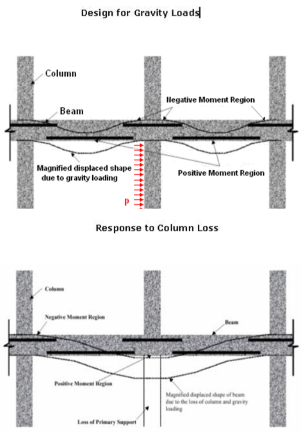
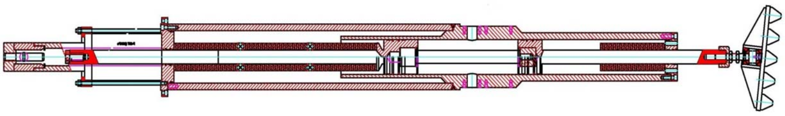
Blast Simulator (Under construction)



(Courtesy of San Diego University)



Actuator prototype design (JRC)

Contents

- I – Introduction (Structures)
- II – ALE formulation (Fluids)
- III – Classical Fluid-Structure Interaction
- IV – Advanced FSI (Failure/Fragmentation)
- V – Further topics and applications

Credits & Acknowledgments

- Structural Dynamics team at JRC Ispra (70's – to date):
  - J. Donea, J.P. Halleux, S. Giuliani, F. Casadei, M. Larcher, G. Solomos, G. Giannopoulos, G. Valsamos, N. Leconte …
- Structural Dynamics team at CEA Saclay (70's – to date):
  - H. Bung, P. Galon, M. Lepareux, V. Faucher, A. Beccantini …
- Contributions from research/academic bodies and industry:
  - UPC (A. Huerta, P. Diez, F. Verdugo), Cachan, CRS4, Lyon, …
  - EDF, Onera, ENEL, Snecma, SamTech …
- Three decades of code development:
  - EURDYN, Castem-PLEXUS / PLEXIS-3C, EUROPLEXUS …
  - Commercial / Academic versions now available from CEA

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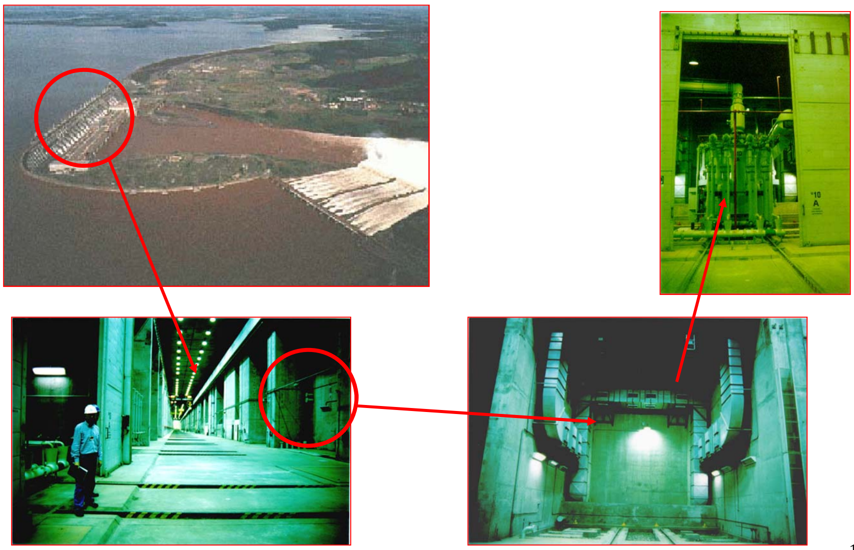
Introductory Example (Courtesy of ENEL-Hydro)



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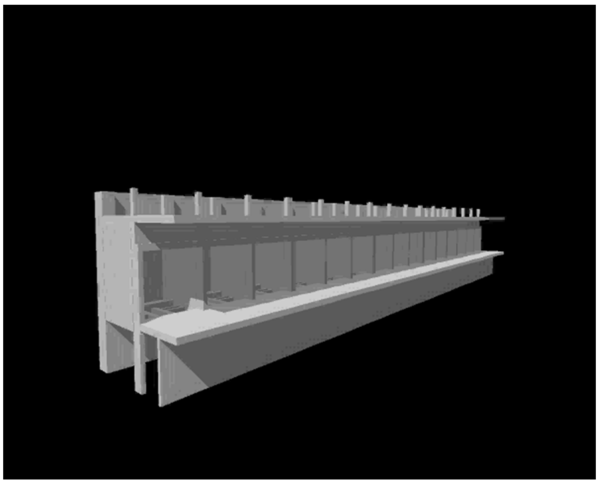
Introductory Example (2)



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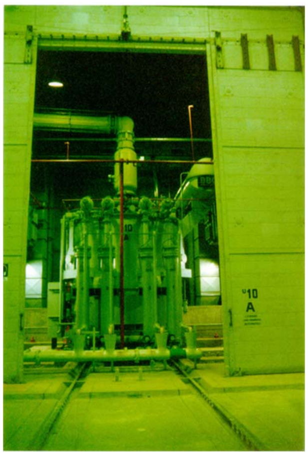
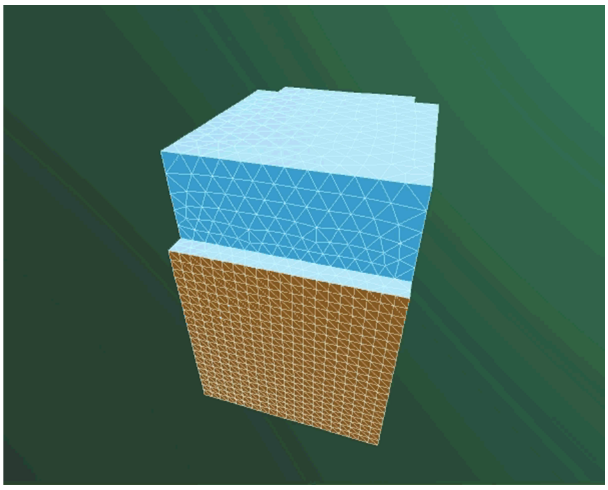
Introductory Example (3)





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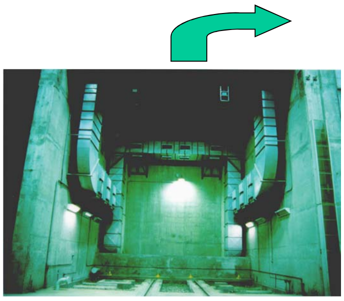
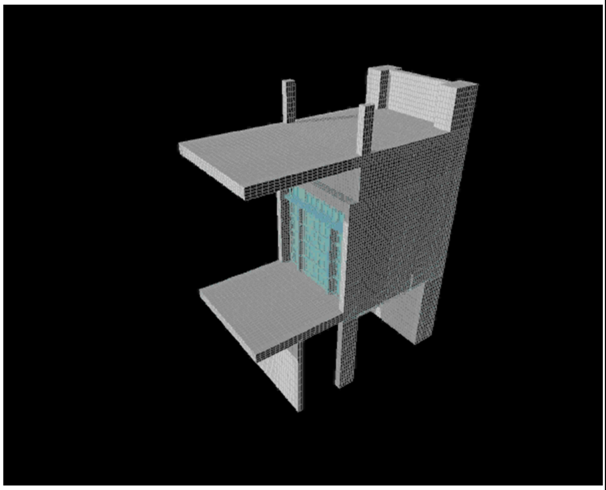
Introductory Example (4)





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Application Spectrum

- Energy (safety issues): nuclear and fossil-fueled plants, electrical devices, chemical plants, pressure vessels, …
- Civil engineering: earthquakes, soil-structure interactions, building vulnerability to terrorist attacks …
- Marine/Offshore: ships and submarines, oil industry, pipelines, cables …
- Transportation: crash, road barriers, tunnels safety, trains/ metro stations and rolling stock…
- etc…

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Goals/Characteristics

- Simulate fast transient dynamic phenomena: explosions, crashes, impacts …
- Short time scale: (typically *ms* to a few *s*) with large frequencies spectrum
- Geometric and material non-linearities: large motions, large strains, plasticity, visco-plasticity, damage …
- Structures and fluids: heterogeneity, interaction phenomena, …
- For reliable solutions, simple and robust numerical methods are needed: direct time integration, explicit schemes …

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Detailed Contents of Part I

- Introductory example of FSI problem
  - ✓ Application spectrum and goals
- Modeling the structural domain
  - ✓ Equilibrium equations
  - ✓ Explicit time integration scheme
- Treatment of essential boundary conditions
  - ✓ The Lagrange multipliers method
  - ✓ Solving the linear system

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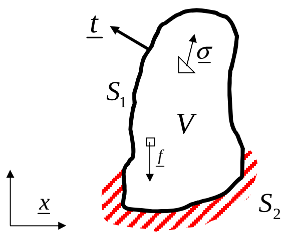
Computational Framework

- Governing equation for structural domain: <u>principle of virtual work</u> (conservation of momentum, i.e. equilibrium in a dynamic sense)

$$\int_V \rho \ddot{\underline{x}} \delta \underline{x} dV + \int_V \underline{\sigma} D(\delta \underline{x}) dV - \int_V \rho \underline{f} \delta \underline{x} dV - \int_{S_1} \underline{t} \delta \underline{x} dS = 0$$

ρ mass density
V current domain
$\underline{x}$ current configuration
$\ddot{\underline{x}}$ accelerations



$\underline{\sigma}$ Cauchy stress
$D()$ spatial derivative operator
$\underline{f}$ volumetric forces per unit mass
$\underline{t}$ boundary surface tractions

Must hold for <u>all</u> variations $\delta \underline{x}$ of configuration (<u>virtual displacements</u>) compatible with <u>essential b.c.s</u> on S_2 .

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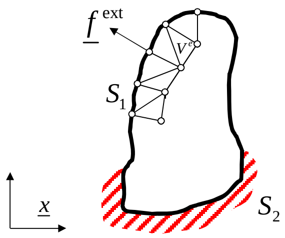
Computational Framework (2)

- This integral form lends itself to direct application of F.E. method. Upon spatial discretization:

$$\underline{M}\ddot{\underline{u}} = \underline{f}^{\text{ext}} - \sum_e \int_{V^e} \underline{B}^T \underline{\sigma} dV$$



$\underline{M}$ Mass matrix
$\underline{u}$ nodal displacement vector
$\underline{f}^{\text{ext}}$ discrete external forces
$\sum_e$ standard F.E. assembly operator
V^e element (e) current volume
$\underline{B}$ matrix of shape functions derivatives

This set of discrete differential equations in time is decoupled by diagonalization (lumping) of mass matrix $\underline{M}$

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$\underline{M}\underline{\ddot{u}} = \underline{f}^{\text{ext}} - \sum_e \int_{V^e} \underline{B}^T \underline{\sigma} dV$

Computational Framework (3)

- Description is Lagrangian: nodes and G.P.s remain always associated with same material point (particle)
- Stress is "true": expressed in fixed reference (but corotational formulation can be useful for beams/plates/shells)
- All RHS terms are known ($\underline{f}^{\text{ext}}, \underline{B}$) or computable: stresses must be obtained via material constitutive law
- Diagonalization of $\underline{M}$ by lumping: $\underline{M}^e = \int_{V^e} \underline{N} \rho dV$ where $\underline{N}$ are the element shape functions
- We work on current configuration: no need to define a reference configuration and no use of (total) deformation

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$\underline{M}\underline{\ddot{u}} = \underline{f}^{\text{ext}} - \sum_e \int_{V^e} \underline{B}^T \underline{\sigma} dV$

Direct Time Integration

- Time integration is achieved via the Central Difference scheme, usually written as:

$$\underline{\dot{u}}^{n+1} = \underline{\dot{u}}^n + \frac{\Delta t}{2}(\underline{\ddot{u}}^n + \underline{\ddot{u}}^{n+1})$$

$$\underline{u}^{n+1} = \underline{u}^n + \Delta t(\underline{\dot{u}}^n + \frac{\Delta t}{2}\underline{\ddot{u}}^n)$$

n stays for time t^n

$n+1$ stays for $t^{n+1} = t^n + \Delta t$

Δt is the time increment

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$$\boxed{\begin{aligned}\underline{\dot{u}}^{n+1} &= \underline{\dot{u}}^{n} + \frac{\Delta t}{2}(\underline{\ddot{u}}^{n} + \underline{\ddot{u}}^{n+1}) \\ \underline{u}^{n+1} &= \underline{u}^{n} + \Delta t(\underline{\dot{u}}^{n} + \frac{\Delta t}{2}\underline{\ddot{u}}^{n})\end{aligned}}$$

Direct Time Integration (2)

- These formulas are a particularization of the well-known Newmark integration formulas:

$$\underline{\dot{u}}^{n+1} = \underline{\dot{u}}^{n} + \Delta t[(1-\gamma)\underline{\ddot{u}}^{n} + \gamma\underline{\ddot{u}}^{n+1}]$$

$$\underline{u}^{n+1} = \underline{u}^{n} + \Delta t\underline{\dot{u}}^{n} + \frac{\Delta t^2}{2}[(1-2\beta)\underline{\ddot{u}}^{n} + 2\beta\underline{\ddot{u}}^{n+1}]$$

written for $\boxed{\gamma = 1/2}$ and $\boxed{\beta = 0}$.

$$\underline{M}\underline{\ddot{u}} = \underline{f}^{\,ext} - \underline{f}^{\,int}$$

- These two equations, plus the equilibrium, can be solved for $\underline{u}$, $\underline{\dot{u}}$, $\underline{\ddot{u}}$ upon step-by-step marching in time.

- This particular choice for β renders the scheme explicit, while the chosen γ ensures no numerical damping.

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$$\boxed{\begin{aligned}\underline{\dot{u}}^{n+1} &= \underline{\dot{u}}^{n} + \frac{\Delta t}{2}(\underline{\ddot{u}}^{n} + \underline{\ddot{u}}^{n+1}) \\ \underline{u}^{n+1} &= \underline{u}^{n} + \Delta t(\underline{\dot{u}}^{n} + \frac{\Delta t}{2}\underline{\ddot{u}}^{n})\end{aligned}}$$

Direct Time Integration (3)

How is the scheme used in practice?

- Introduce a mid-step velocity: $\underline{v}^{n+1/2} \doteq \underline{\dot{u}}^{n} + \frac{\Delta t}{2}\underline{\ddot{u}}^{n}$

  which transforms configuration *n* into *n+1* over the step.

- The second equation becomes:

$$\underline{u}^{n+1} = \underline{u}^{n} + \Delta t \cdot \underline{v}^{n+1/2}$$

- Carry on mid-step velocities rather than full-step ones. The first equation becomes:

$$\underline{v}^{n+3/2} = \underline{v}^{n+1/2} + \Delta t \cdot \underline{\ddot{u}}^{n+1}$$

$$\underline{v}^{n+3/2} \doteq \underline{\dot{u}}^{n+1} + \frac{\Delta t}{2}\underline{\ddot{u}}^{n+1} = \underline{\dot{u}}^{n} + \frac{\Delta t}{2}\underline{\ddot{u}}^{n} + \frac{\Delta t}{2}\underline{\ddot{u}}^{n+1} + \frac{\Delta t}{2}\underline{\ddot{u}}^{n+1} = \underline{v}^{n+1/2} + \Delta t \cdot \underline{\ddot{u}}^{n+1}$$

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Direct Time Integration (4)

- The <u>final algorithm</u> reads:

How does one obtain $\underline{\sigma}^{e(n+1)}$? (see below)

$$\underline{u}^{n+1} = \underline{u}^{n} + \Delta t \cdot \underline{v}^{n+1/2}$$

$$\underline{\ddot{u}}^{n+1} = \underline{M}^{-1}\left(\underline{f}^{\,\mathrm{ext}(n+1)} - \sum_e \int_{V^{e(n+1)}} \underline{B}^T \underline{\sigma}^{e(n+1)} dV\right)$$

$$\underline{v}^{n+3/2} = \underline{v}^{n+1/2} + \Delta t \cdot \underline{\ddot{u}}^{n+1}$$

A <u>new configuration</u> is obtained first. On this known configuration, <u>equilibrium</u> is enforced. The <u>new mid-step velocity</u> is obtained last.

- This scheme is <u>explicit</u>.

- If Δt varies in time, the only change is in the third equation, which becomes:

$$\underline{v}^{n+3/2} = \underline{v}^{n+1/2} + \frac{\Delta t^{n} + \Delta t^{n+1}}{2} \cdot \underline{\ddot{u}}^{n+1}$$

with:

$$\Delta t^{n} \equiv t^{n+1} - t^{n}$$
$$\Delta t^{n+1} \equiv t^{n+2} - t^{n+1}$$

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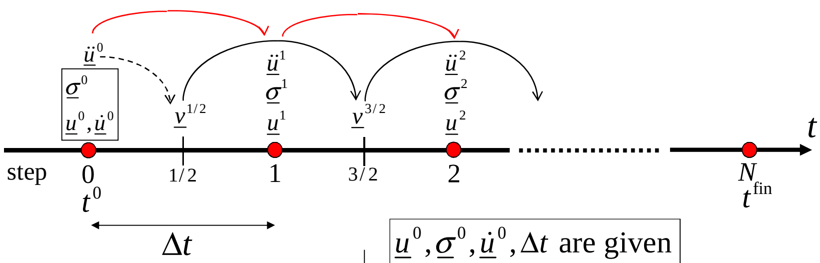
Scheme start-up and marching



$\boxed{\underline{u}^0, \underline{\sigma}^0, \underline{\dot{u}}^0, \Delta t \text{ are given}}$

$n = -1$

$\underline{\ddot{u}}^0 = \underline{M}^{-1}(\underline{f}^{\,\mathrm{ext}} - \underline{f}^{\,\mathrm{int}})^0$

$\underline{v}^{1/2} = \underline{\dot{u}}^0 + (\Delta t/2) \cdot \underline{\ddot{u}}^0$

$n \leftarrow n+1$

$\underline{u}^1 = \underline{u}^0 + \Delta t \cdot \underline{v}^{1/2}$

$\underline{\ddot{u}}^1 = \underline{M}^{-1}(\underline{f}^{\,\mathrm{ext}} - \underline{f}^{\,\mathrm{int}})^1$

t $\quad \underline{v}^{3/2} = \underline{v}^{1/2} + \Delta t \cdot \underline{\ddot{u}}^1$

etc.

$$\underline{u}^{n+1} = \underline{u}^{n} + \Delta t \cdot \underline{v}^{n+1/2}$$
$$\underline{\ddot{u}}^{n+1} = \underline{M}^{-1}(\underline{f}^{\,\mathrm{ext}} - \underline{f}^{\,\mathrm{int}})^{n+1}$$
$$\underline{v}^{n+3/2} = \underline{v}^{n+1/2} + \Delta t \cdot \underline{\ddot{u}}^{n+1}$$

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Scheme start-up and marching (2)

- For practical reasons, the code also computes the <u>full-step</u> velocities:

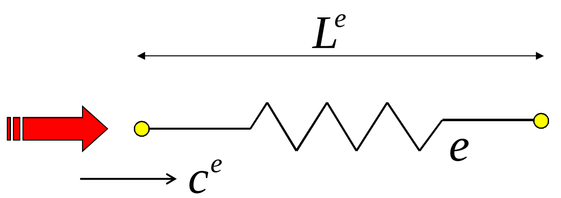
$$\dot{\underline{u}}^{n+1} = \underline{v}^{n+1/2} + \frac{\Delta t}{2}\ddot{\underline{u}}^{n+1}$$

- These are the velocities printed out in the listing and visualized in post-processing

- However, the <u>fundamental</u> quantity in the time integration scheme is the <u>mid-step</u> velocity!

Integration Scheme Characteristics

- Central difference scheme is <u>second-order accurate</u> and introduces <u>no numerical damping</u>
- However, it is <u>conditionally stable</u> (Courant):



$$\boxed{\Delta t^e_{\text{stab}} \approx L^e / c^e}$$

$\Delta t^e = \varphi \cdot \Delta t^e_{\text{stab}}$

(with $\varphi < 1$)

- In highly non-linear cases, small steps are needed even with unconditionally stable schemes, to get good accuracy

Integration Scheme Characteristics (2)

- Spectral analysis shows that the central difference scheme tends to produce frequencies slightly higher than physical ones. Same effect is obtained using a consistent mass matrix.
- However, use of a lumped mass matrix tends to reduce frequency values.
- Therefore, combination of CD time integrator with a lumped mass matrix gives optimal numerical precision.
- This is a remarkable result, since the final equations are completely decoupled: contrary to classical FE method, there are no matrices to assemble and no need for system solvers (except for treatment of essential BCs).

Integration Scheme Characteristics (3)

- See: S.W.Key, "*Transient Response by Time Integration*".

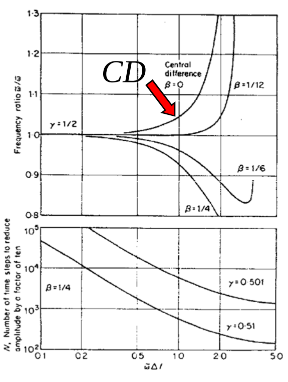


FIG. 6. The frequency response for $\gamma = 0{\cdot}5$ and damping for $\beta = 0{\cdot}25$ for the Newmark β time integrator.

Scheme effect on frequency

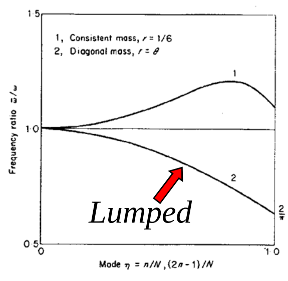


FIG. 2. The ratio of approximate frequency, $\bar{\omega}$, to true frequency, ω, versus non-dimensional wave number for a piecewise linear spatial approximation.

Mass matrix effect on frequency

Stress Update

- To solve the equilibrium equation for the new accelerations:

$$\ddot{\underline{u}}^{n+1} = \underline{M}^{-1}(\underline{f}^{\text{ext}(n+1)} - \sum_e \int_{V^{e(n+1)}} \underline{B}^T \underline{\sigma}^{e(n+1)} dV)$$

one needs the new stress $\underline{\sigma}^{e(n+1)}$.

- In general one can formally write:

$$\underline{\sigma}^{n+1} = \underline{\sigma}^n + \Delta\underline{\sigma}$$

$$\Delta\underline{\sigma} = H(\underline{\sigma}^n, \Delta\underline{\varepsilon}, \underline{p}, \dot{\underline{\varepsilon}}, \ldots) \quad \text{(Rate form)}$$

H Constitutive law

$\Delta\underline{\sigma}$ stress increment over the step

$\Delta\underline{\varepsilon}$ strain increment over the step

$\underline{p}$ hardening parameters (e.g. plasticity)

$\dot{\underline{\varepsilon}}$ strain rate (e.g. viscous behaviour)

- Note that the total deformation $\underline{\varepsilon}$ does not appear anywhere and is not used in the process.

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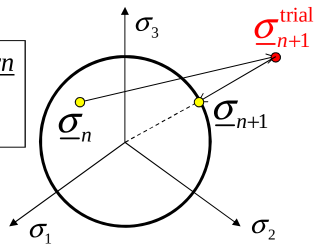
Elasto-plastic material

As an example of non-linear material behaviour consider the important case of metal plasticity:

- Rate-independent deviatoric plasticity model with Von Mises yield criterion:

Radial return method (Wilkins)



- “Trial” stress (elastic):

$$\underline{\sigma}_{n+1}^{\text{trial}} = \underline{\sigma}_n + \underline{\underline{C}} \cdot \Delta\underline{\varepsilon}$$

- If trial stress lies outside yield surface, perform radial return onto (current) yield surface. No iterations!

How does one compute $\Delta\underline{\varepsilon}$ from displacement increments (or velocities) in the presence of *geometrical non-linearities* (large strains and large motions, in particular large rotations?)

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Geometric non-linearities

A large-displacement large-strain formulation is adopted for full generality. For continuum-like FE:

- Compute spatial velocity gradient:

$$\underline{L} = \partial \underline{\dot{x}} / \partial \underline{x}$$

- Use additive decomposition to separate instantaneous deformation (symmetric) from rotation (antisymmetric part):

$$\underline{L} = \underline{D} + \underline{W}$$

$$\underline{D} = \frac{1}{2}(\underline{L} + \underline{L}^T) \quad (\textit{stretching} \text{ i.e. rate of deformation})$$

$$\underline{W} = \frac{1}{2}(\underline{L} - \underline{L}^T) \quad (\textit{spin} \text{ i.e. rate of rotation})$$

- We obtain then:

$$\boxed{\underline{\dot{\varepsilon}} = \underline{D} \qquad ; \qquad \Delta\underline{\varepsilon} = \underline{D} \cdot \Delta t}$$

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Geometric non-linearities (2)

For a continuum element the state of stress of interest to us, Cauchy stress $\underline{\sigma}$, is referred to a fixed frame in space. Consequently, its time derivative is not invariant with respect to rotation: $\underline{\dot{\sigma}}$ is *not objective*.

- An objective rate of stress $\underline{\hat{\sigma}}$ can be obtained under the form: $\underline{\hat{\sigma}} = \underline{\dot{\sigma}} - \underline{A}\underline{\sigma} + \underline{\sigma}\underline{A}$ where $\underline{A}$ is an appropriate vorticity matrix.

- In the Zaremba-Jaumann-Noll formulation: $\underline{A} \doteq \underline{W}$

  (other choices are possible, e.g. Green-Naghdi).

- However, the above considerations are valid only in an infinitesimal sense, while we need to use finite increments.

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Geometric non-linearities (3)

Set up following <u>incrementally objective</u> scheme to update the Cauchy stress (2D case for simplicity) in <u>three phases</u>:

- Let α be the angle of rotation over Δt and let $\theta = \alpha / 2$

$$\tan\theta = (\Delta t / 2) \cdot W_{12}^{n+1/2}$$

1. Apply <u>first half</u> of the rotation increment:

$$\underline{\sigma}^{n*} = \underline{R}\,\underline{\sigma}^{n}\,\underline{R}^{T} \quad \text{with} \quad \underline{R} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

2. Apply the <u>constitutive equation:</u>

$$\boxed{\underline{\sigma}^{(n+1)*} = \underline{\sigma}^{n*} + \underline{\underline{C}} \cdot \Delta t \cdot \underline{D}^{n+1/2}}$$

3. Apply <u>second half</u> of the rotation increment:

$$\underline{\sigma}^{n+1} = \underline{R}\,\underline{\sigma}^{(n+1)*}\,\underline{R}^{T}$$

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Geometric non-linearities (4)

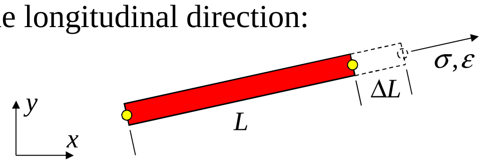
For <u>structural elements</u> (bars, beams, shells) use <u>co-rotational</u> formulation:

- The stress is measured in a reference frame that <u>rotates</u> with the element
- This greatly simplifies the stress increment procedure: the stress can be <u>incremented directly</u> by applying the constitutive law.

Example (bar element). In the longitudinal direction:

$$\boxed{\Delta\varepsilon = \frac{\Delta L}{L}} \quad \rightarrow \quad \Delta\sigma$$

$$\boxed{\sum \Delta\varepsilon} = \sum \frac{\Delta L}{L} \simeq \int_{L_0}^{L} \frac{dL}{L} = \boxed{\ln \frac{L}{L_0}}$$



(<u>Natural</u> or <u>logarithmic</u> strain)

A <u>small-strain</u> formulation would be:

$$\boxed{\Delta\varepsilon = \frac{\Delta L}{L_0}} \quad \rightarrow \quad \boxed{\sum \Delta\varepsilon = \frac{L - L_0}{L_0}}$$

(<u>Engineering</u> strain)

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Advantages of the Method

- Transient dynamic problem: find $\underline{\sigma}$ on new configuration $\underline{x}$ (known) from $\underline{\sigma}^{\text{old}}$ and deformation process between $\underline{x}^{\text{old}}$ and $\underline{x}$
- Compare implicit methods: find $\underline{\sigma}$ and $\underline{x}$ simultaneously, typically by iterative procedures and convergence criteria.
- The proposed method is particularly simple for complex non-linear problems, hence very robust.
- Direct application of virtual work principle plus second-order accurate time integration scheme, guarantee high accuracy of numerical results.

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Checking the Solution Quality

- The quality of the obtained solution can be checked globally by computing at each time step the *energy balance*.
- Initially, set: $W_0^{\text{ext}} \doteq E_0^{\text{int}} + E_0^{\text{kin}}$
- At any time, the balance error can be computed as:

$$\varepsilon = \frac{W^{\text{ext}} - (E^{\text{int}} + E^{\text{kin}})}{W^{\text{ext}}}$$ or perhaps better: $$\boxed{\varepsilon = \frac{W^{\text{ext}} - (E^{\text{int}} + E^{\text{kin}})}{\max\left(\left|W^{\text{ext}}\right|, \left|W_0^{\text{ext}}\right|\right)}}$$

- This error indicator is used *a posteriori* in order to check the previously obtained solution and must not be confused with convergence parameters typical of iterative approaches

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Essential Boundary Conditions

Essential conditions are imposed via Lagrange multipliers.

- Assume a linear set of constraints on the velocities:

$$\underline{C}\,\underline{v} = \underline{b}$$

- Both $\underline{C}$ and $\underline{b}$ are known, and can be function of time.
- The equilibrium equations for the subset of d.o.f.s concerned become, introducing unknown reactions $\underline{r}$:

$$\underline{m}\,\underline{a} = \underline{f}^{\mathrm{e}} - \underline{f}^{\mathrm{i}} + \underline{r}$$

- Without loss of generality, the unknown reactions can be expressed via a vector $\underline{\lambda}$ of Lagrange multipliers:

$$\underline{r} = \underline{C}^T \underline{\lambda}$$

Finding the Lagrange Multipliers

- Replacing into the equilibrium equations yields:

$$\underline{m}\,\underline{a} = \underline{f}^{\mathrm{e}} - \underline{f}^{\mathrm{i}} + \underline{C}^T \underline{\lambda}$$

- Multiplying both members by $\underline{C}\,\underline{m}^{-1}$ gives:

$$\underline{C}\,\underline{a} = \underline{C}\,\underline{m}^{-1}(\underline{f}^{\mathrm{e}} - \underline{f}^{\mathrm{i}}) + \underbrace{\underline{C}\,\underline{m}^{-1}\underline{C}^T}_{\underline{B}^*}\,\underline{\lambda}$$

Matrix of connections

- The Lagrange multipliers are obtained symbolically from:

$$\underline{B}^* \underline{\lambda} = \underline{C}\,\underline{a} - \underline{C}\,\underline{m}^{-1}(\underline{f}^{\mathrm{e}} - \underline{f}^{\mathrm{i}})$$

- To obtain $\underline{\lambda}$, we must first express the term $\underline{C}\,\underline{a}$ as a function of known quantities, by using the constraint and the time integration scheme.

Finding the Lagrange Multipliers (2)

- The CD scheme for the velocity and <u>constant</u> Δt is:
  $\underline{v}^{n+3/2} = \underline{v}^{n+1/2} + \Delta t \cdot \underline{a}^{n+1}$
- Substituting this into the constraint $\underline{C}\,\underline{v} = \underline{b}$ gives:
  $\underline{C}\,\underline{v}^{n+3/2} = \underline{C}\,\underline{v}^{n+1/2} + \Delta t \cdot \underline{C}\,\underline{a}^{n+1} = \underline{b}$
- From this we obtain:
  $\underline{C}\,\underline{a} = \frac{1}{\Delta t}(\underline{b} - \underline{C}\,\underline{v}^{n+1/2}) = \frac{1}{\bar{\gamma}}(\underline{b} - \underline{C}\,\underline{v}^{n+1/2})$ having posed: $\boxed{\bar{\gamma} = \Delta t}$
- For a <u>variable</u> Δt in time, one has simply:
  $\boxed{\bar{\gamma} = \frac{\Delta t^n + \Delta t^{n+1}}{2}}$

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Finding the Lagrange Multipliers (3)

- Summarizing, the Lagrange multipliers $\underline{\lambda}$ are obtained by solving the linear algebraic system:

  $$\underline{B}^*\underline{\lambda} = \underline{w}$$

  We obtain one multiplier for each imposed constraint

  where the known terms are given by:

  $\underline{B}^* \equiv \underline{C}\,\underline{m}^{-1}\underline{C}^T$ and $\underline{w} \equiv \frac{1}{\bar{\gamma}}(\underline{b} - \underline{C}\,\underline{v}^{n+1/2}) - \underline{C}\,\underline{m}^{-1}(\underline{f}^{\,e} - \underline{f}^{\,i})$

  $\bar{\gamma} = \Delta t$ for constant Δt

  $\bar{\gamma} = (\Delta t^n + \Delta t^{n+1})/2$ for variable Δt in time
- Finally we compute the <u>reactions</u>:

  $$\underline{r} = \underline{C}^T\underline{\lambda}$$

  We obtain one reaction for each constrained dof

  and add them to the other known external forces.
- This is the <u>only implicit part</u> of the whole method.

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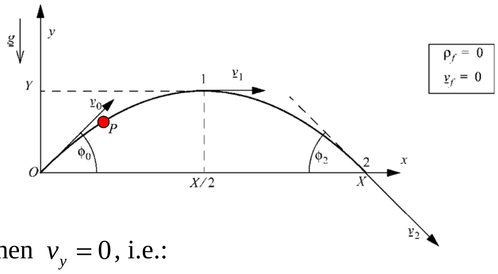
Exercise 0 – Ideal ballistics

- Motion in vacuum is analytical:

$$v_x = v_{0x} = v_0 \cos\phi_0$$
$$v_y = v_{0y} - gt = v_0 \sin\phi_0 - gt$$



- The trajectory is a parabola:

$$\phi_2 = \phi_0 \quad \text{and} \quad \|\underline{v}_2\| = \|\underline{v}_0\|$$

- Time to reach highest point is when $v_y = 0$, i.e.:

$$t_1 = \frac{v_{0y}}{g} = \frac{v_0}{g}\sin\phi_0$$

- By symmetry, time to impact is twice as long:

$$t_2 = 2t_1 = 2\frac{v_{0y}}{g} = 2\frac{v_0}{g}\sin\phi_0$$

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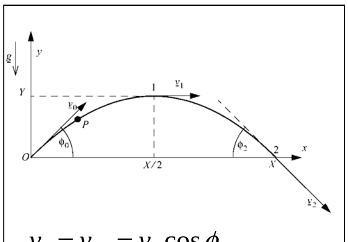
Exercise 0 – Ideal ballistics (2)



$$v_x = v_{0x} = v_0 \cos\phi_0$$
$$v_y = v_{0y} - gt = v_0 \sin\phi_0 - gt$$
$$t_1 = \frac{v_{0y}}{g} = \frac{v_0}{g}\sin\phi_0$$
$$t_2 = 2t_1 = 2\frac{v_{0y}}{g} = 2\frac{v_0}{g}\sin\phi_0$$

- The range is therefore:

$$X = v_x t_2 = 2v_{0x}\frac{v_{0y}}{g} = 2\frac{v_0^2}{g}\sin\phi_0\cos\phi_0 = \frac{v_0^2}{g}\sin(2\phi_0)$$

- Max range is when shooting at 45º:

$$X_{\max}(\phi_0) = \frac{v_0^2}{g} \quad \text{for} \quad \sin(2\phi_0) = 1 \Leftrightarrow \phi_0 = \pi/4$$

- The position at the generic time is:

$$x(t) = v_{0x}t = v_0 t\cos\phi_0$$
$$y(t) = v_{0y}t - g\frac{t^2}{2} = v_0 t\sin\phi_0 - g\frac{t^2}{2}$$

- Max elevation depends only on v_{0y}:

$$Y = y_{\max} = y(t_1) = \frac{v_{0y}^2}{g} - \frac{g}{2}\frac{v_{0y}^2}{g^2} = \frac{v_{0y}^2}{2g} = \frac{v_0^2}{2g}\sin^2\phi_0$$

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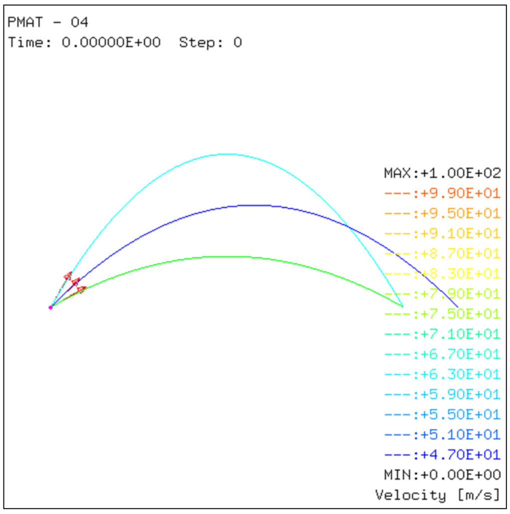
Exercise 0 – Ideal ballistics (3)

- Study motion of projectile as a function of the shooting angle:

$$v_0 = 100$$
$$\phi_0 = 30°, 45°, 60°$$

- Computed vs. analytical positions:



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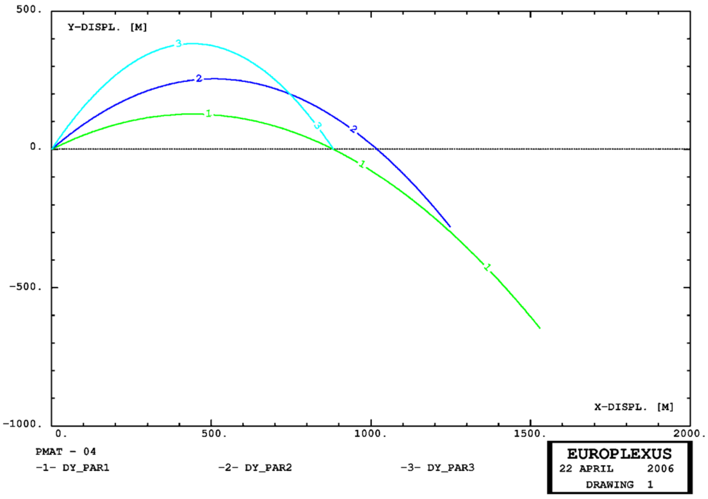
Exercise 0 – Ideal ballistics (4)

- Computed trajectories:



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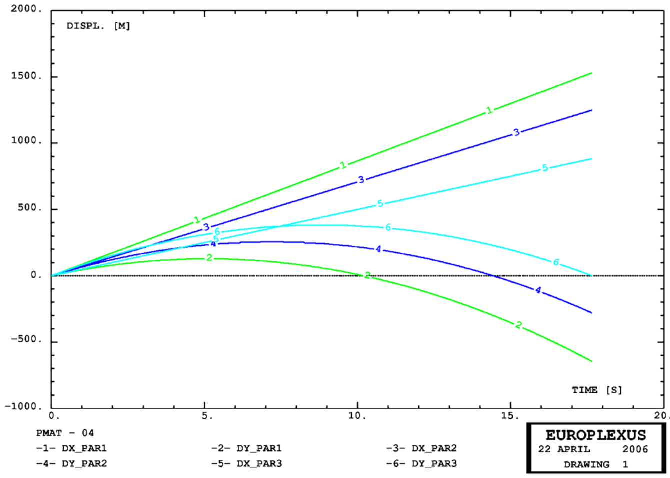
Exercise 0 – Ideal ballistics (5)

- Computed displacement components:



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Exercise 0 – Ideal ballistics (6)

- Influence of time discretization:

Case	Description	Flight time t_2	Range X	N. of steps
PMAT01	$\phi_0 = 30°$	10.197	883.1	1000
PMAT01B	$\phi_0 = 30°$	10.197	883.1	100
PMAT01C	$\phi_0 = 30°$	10.197	883.1	10
PMAT01D	$\phi_0 = 30°$	10.197	883.1	2
PMAT01E	$\phi_0 = 30°$	10.197	883.1	1
PMAT02	$\phi_0 = 45°$	14.421	1019.7	1000
PMAT03	$\phi_0 = 60°$	17.662	883.1	1000
PMAT04	$\phi_0 = 30°, 45°, 60°$	10.197, 14.421, 17.662	883.1, 1019.7	1000

- No stability restraints (no wave propagation)
- Analytical precision

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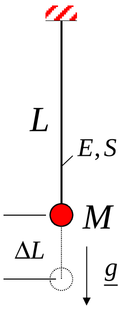
Exercise 1 – Suspended mass

- Single-element discrete model: check vs. (linear) analytical solution
- Explain possible reasons for observed discrepancies
- Try out different values: e.g. gravity 1000 times smaller
- Discuss multi-element discrete model



$L = 1$ m
$M = 100$ kg
$g = 10$ m/s^2
$S = 2.5\times10^{-8}$ m^2
$E = 2\times10^{11}$ Pa
$\rho = 8000$ kg/m^3
$\nu = 0$

- Replace gravity by initial velocity and discuss effects of structure modeling: A) as a bar, B) as a cable …

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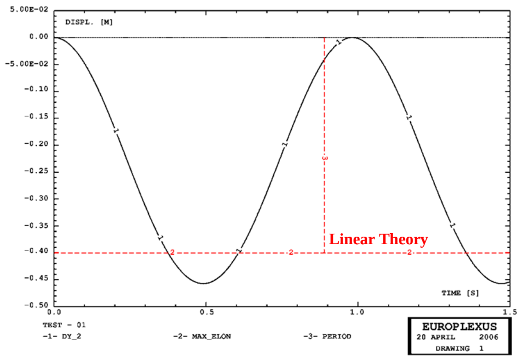
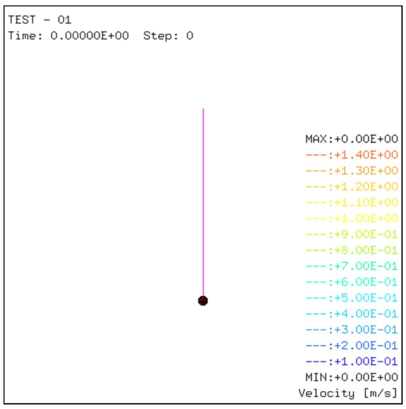
Exercise 1 – Suspended mass (2)

- **TEST01** : 1 element of type FUN2 (cable), no resistance to bending. Use FUNE material (no resistance to compression)





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Exercise 1 – Suspended mass (3)

- EUROPLEXUS input file:

```
TEST - 01
*-----------------------------------------------------------------------
ECHO
*CONV win
*----------------------------------------------------------Problem type
CPLA LAGR
*----------------------------------------------------------Dimensioning
DIME
 PT2L 2 FUN2 1 PMAT 1 ZONE 2
 TABL 1 2
 FORC 1
TERM
*--------------------------------------------------------------Geometry
GEOM LIBR POIN 2 FUN2 1 PMAT 1 TERM
 0 0  0 -1
 1 2
 2
*-----------------------------------------------Geometrical complements
COMP EPAI 2.5E-8 LECT 1 TERM
*---------------------------------------------------------Material data
MATE FUNE RO 8000. YOUN 2.0E11 NU 0.0 ELAS 2.0E11 ERUP 1.0E0
        TRAC 1 2.0E11 1.E0
        LECT 1 TERM
    MASS 100.0 LECT 2 TERM
*---------------------------------------------------Boundary conditions
LINK COUP
    BLOQ 2 LECT 1 TERM
*---------------------------------------------------------Applied loads
CHAR 1 FACT 2 FORC 2 -1.E3 LECT 2 TERM
      TABL 2 0.0 1.0 10.0 1.0
*---------------------------------------------------------------Outputs
ECRI DEPL VITE CONT ECRO TFREQ 0.5
    FICH ALIC TEMP FREQ 20
            POIN LECT 1 2 TERM
            ELEM LECT 1 TERM
*---------------------------------------------------------------Options
OPTI PAS UTIL NOTEST
*-------------------------------------------------Transient calculation
CALCUL TINI 0. TEND 1.5 PASF 0.1E-3
```

```
*=====================================================
=======POST-TREATMENT
SUIT
Post-treatment
ECHO
RESU ALIC TEMP GARD PSCR
SORT GRAP
AXTE 1.0 'Time [s]'
*--------------------------------------------------Curve definitions
COUR 1 'dy_2'    DEPL COMP 2 NOEU LECT  2 TERM
COUR 2 'sg_1'    CONT COMP 1 ELEM LECT  1 TERM
COUR 3 'fe_2'    FORC COMP 2 NOEU LECT  2 TERM
COUR 4 'fe_1'    FORC COMP 2 NOEU LECT  1 TERM
DCOU 6 'Max_elon' 2
     0.0 -0.4
     1.5 -0.4
DCOU 7 'Period' 2
     0.889 0.0
     0.889 -0.4
*-------------------------------------------------------------Plots
trac 1 6 7 axes 1.0 'DISPL. [M]' yzer
        colo noir roug roug
        dash 0    2    2
trac 2 axes 1.0 'CONTR. [PA]' yzer
trac 3 4 axes 1.0 'FORCE [N]' yzer
list 1 axes 1.0 'DISPL. [M]'
*----------------------------------------------Results qualification
QUAL DEPL COMP 2 LECT   2 TERM REFE -4.53636E-1  TOLE 1.E-2
   CONT COMP 1 LECT   1 TERM REFE  7.48171E+10 TOLE 1.E-2
*=====================================================
======================
FIN
```

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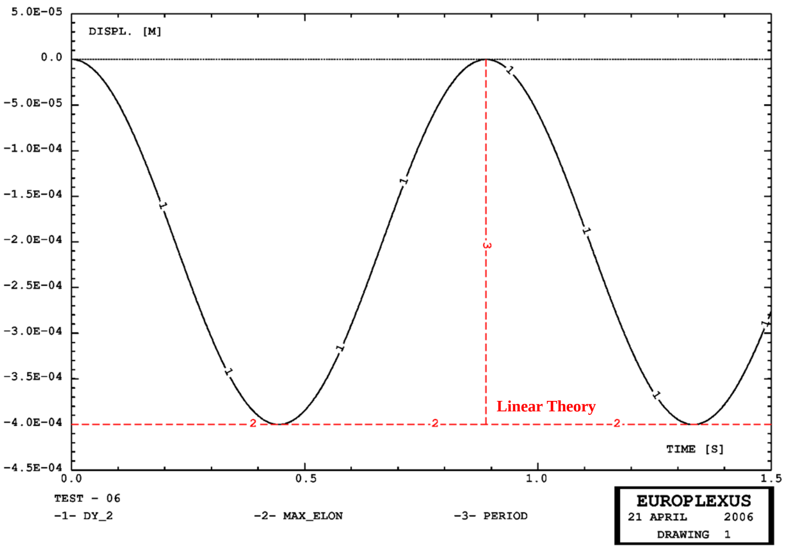
Exercise 1 – Suspended mass (4)

- **TEST06** : applied load 1000 times smaller (small-strain)



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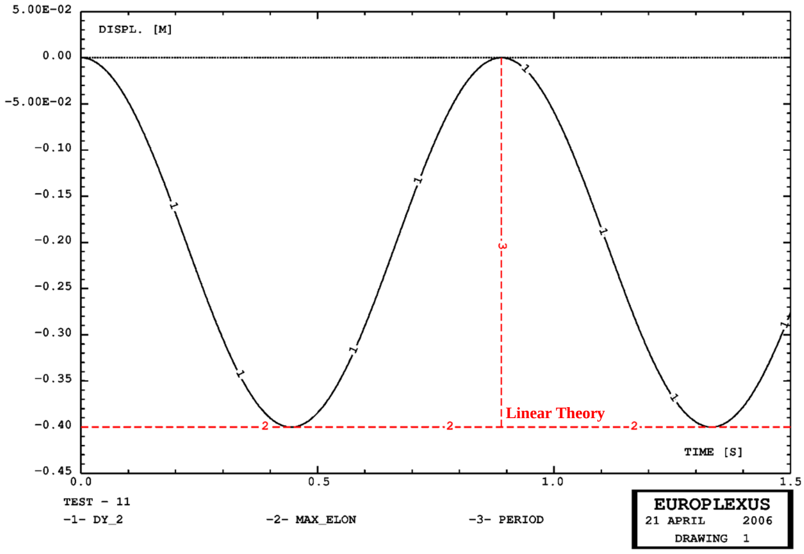
Exercise 1 – Suspended mass (5)

- **TEST11** : standard load but small-strain option (OPTI EDSS)



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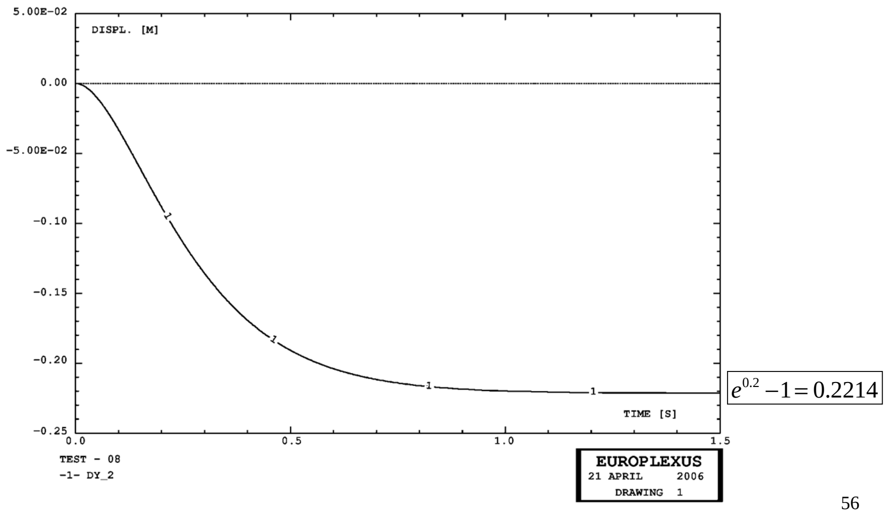
Exercise 1 – Suspended mass (6)

- **TEST08** : “quasi-static” option (OPTI QUAS STAT …)



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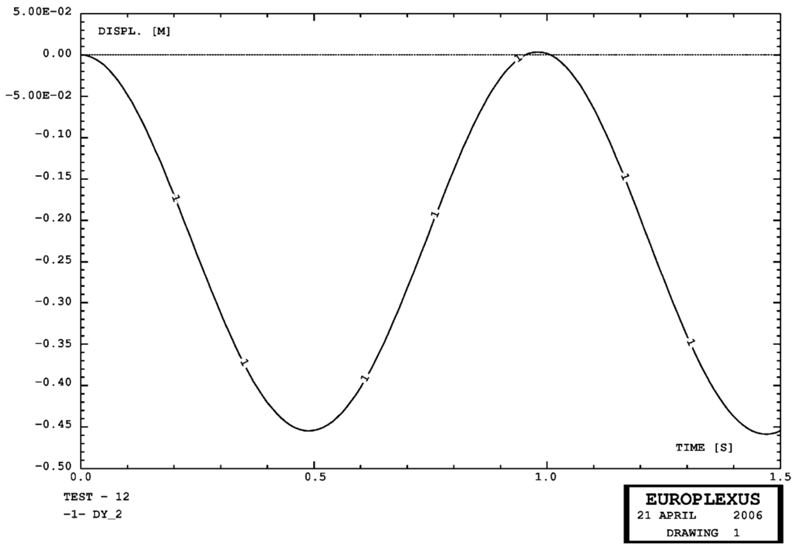
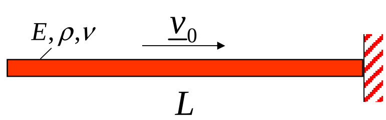
Exercise 1 – Suspended mass (7)

- **TEST12** : time increment 50 times the critical value



Exercise 2 – Wave propagation

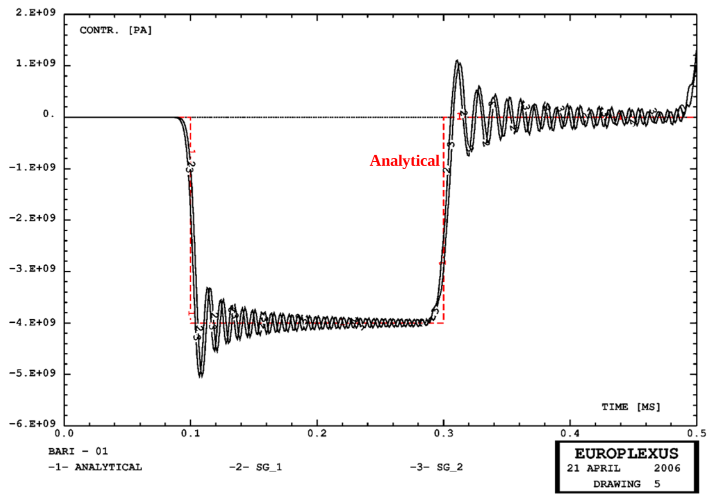
- Obtain 1D analytical solution
- Discuss numerical solution
- Why was cross-section not specified?
- Study effect of time increment
- Study effect of Poisson's ratio …



$L = 1$ m
$v_0 = 100$ m/s
$E = 2\times10^{11}$ Pa
$\rho = 8000$ kg/m^3
$\nu = 0.3$

Exercise 2 – Wave propagation (2)

- **BARI01** : 100 elements of type FUN2 (cable), VM23 material (traction/compression), small-strain option (OPTI EDSS)



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Exercise 2 – Wave propagation (3)

- EUROPLEXUS input file:

```
BARI - 01
*--------------------------------------------------------------------
ECHO
*CONV win
CAST mesh
*---------------------------------------------------------Problem type
CPLA LAGR LAGC
*---------------------------------------------------------Dimensioning
DIME
 PT2L 102 FUN2 100 PMAT 1 ZONE 2
 BLOQ 2
 TABL 1 2
 FORC 1
 IMPA 1 PSIM 1
TERM
*-------------------------------------------------------------Geometry
GEOM FUN2 bar PMAT obst TERM
*----------------------------------------------Geometrical complements
COMP EPAI 2.5E-8 LECT bar TERM
*--------------------------------------------------------Material data
MATE VM23 RO 8000. YOUN 2.0E11 NU 0.0 ELAS 2.0E11
      TRAC 1 2.0E11 1.E0
      LECT bar TERM
   MASS 1.0 LECT obst TERM
*-------------------------------------------------Boundary conditions
LIAI FREQ 1
   BLOQ 12 LECT obst TERM
   IMPA DDL 1 COTE -1
      PROJ LECT obst TERM
      CIBL LECT p2   TERM
*---------------------------------------------------Initial conditions
INIT VITE 1 100.0 LECT bar TERM
*--------------------------------------------------------------Outputs
ECRI DEPL VITE CONT ECRO TFREQ 0.1E-3
   FICH ALIC TEMP FREQ 1
         POIN LECT p3 p2 TERM
         ELEM LECT e1 e2 TERM
*--------------------------------------------------------------Options
OPTI PAS UTIL NOTEST
   LOG 1
   EDSS
*-----------------------------------------------Transient calculation
CALCUL TINI 0. TEND 0.5E-3 PASF 0.1E-5
```

```
*=================================================================
=======POST-TREATMENT
SUIT
Post-treatment
ECHO
RESU ALIC TEMP GARD PSCR
SORT GRAP
AXTE 1000.0 'Time [ms]'
*---------------------------------------------------Curve definitions
COUR 1 'dx_2'    DEPL COMP 1 NOEU LECT p2 TERM
COUR 2 'dx_3'    DEPL COMP 1 NOEU LECT p3 TERM
COUR 3 'sg_1'    CONT COMP 1 ELEM LECT e1 TERM
COUR 4 'sg_2'    CONT COMP 1 ELEM LECT e2 TERM
DCOU 5 'Analytical' 6
     0.0    0.0
     0.1E-3 0.0
     0.1E-3 -4.E9
     0.3E-3 -4.E9
     0.3E-3 0.0
     0.5E-3 0.0
*-------------------------------------------------------------Plots
trac 1 axes 1.0 'DISPL. [M]'
trac 2 axes 1.0 'DISPL. [M]'
trac 3 axes 1.0 'CONTR. [PA]'
trac 4 axes 1.0 'CONTR. [PA]'
trac 5 3 4 axes 1.0 'CONTR. [PA]' yzer
        colo roug noir noir
        dash 2    0    0
*---------------------------------------------Results qualification
QUAL DEPL COMP 1 LECT  p2 TERM REFE -9.76759E-3  TOLE 1.E-2
   DEPL COMP 1 LECT  p3 TERM REFE -9.86359E-3  TOLE 1.E-2
   CONT COMP 1 LECT  e1 TERM REFE  1.00128E+9  TOLE 1.E-2
   CONT COMP 1 LECT  e2 TERM REFE  1.31288E+9  TOLE 1.E-2
*=================================================================
=====================
FIN
```

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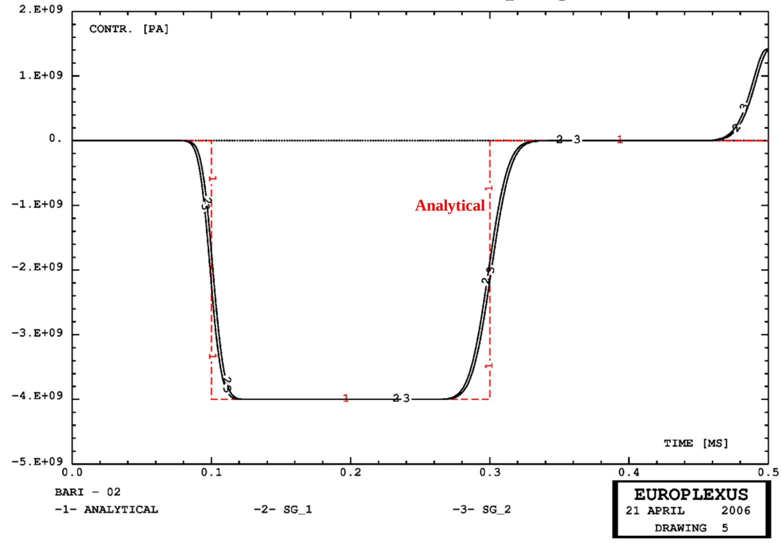
Exercise 2 – Wave propagation (4)

- **BARI02** : 10 % of critical damping added



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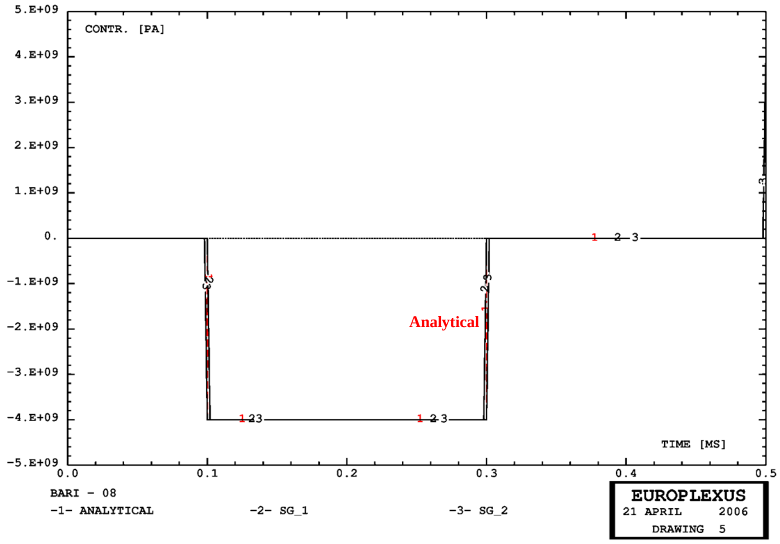
Exercise 2 – Wave propagation (5)

- **BARI08** : use critical time increment



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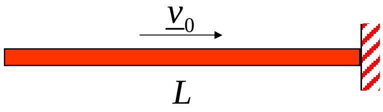
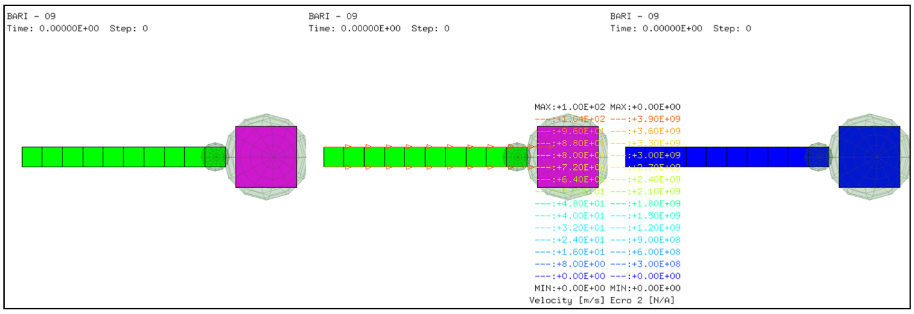
Exercise 2 – Wave propagation (6)

- **BARI09** : 2-D geometry (Q42L) and pinballs for contacts:





Geometry *Velocities* *Von Mises*

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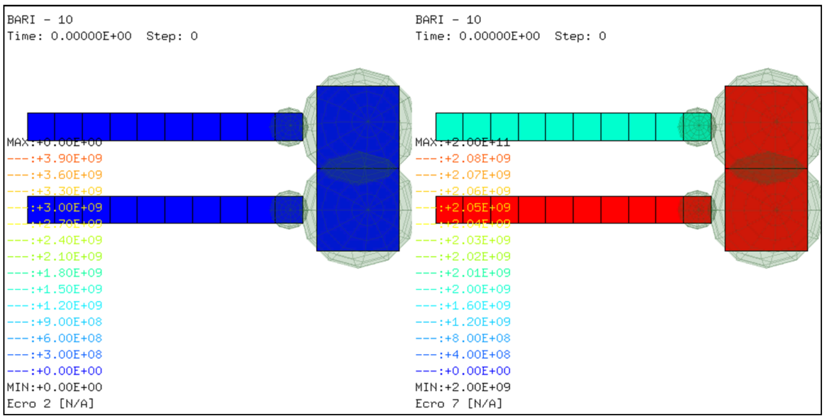
Exercise 2 – Wave propagation (7)

- **BARI10** : Compare elastic (bottom) and <u>elasto-plastic</u> (top) solutions:



Von Mises *Yield Limit*

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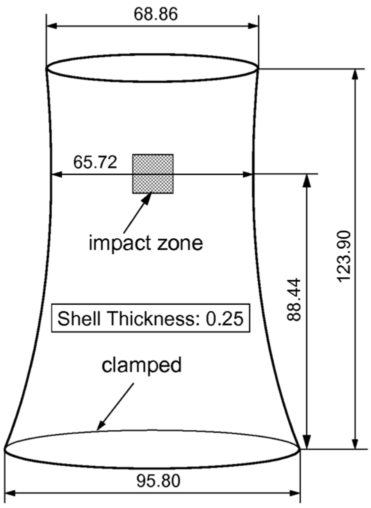
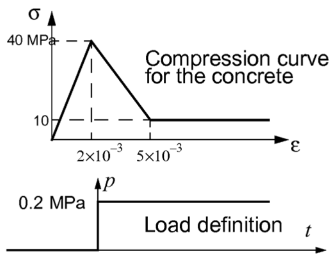
Exercise 3 – Impact on Cooling Tower

- Problem definition:



Layer N.	φ_L	Material
1	0.1	concrete
2	0.0025	steel
3	0.795	concrete
4	0.0025	steel
5	0.1	concrete



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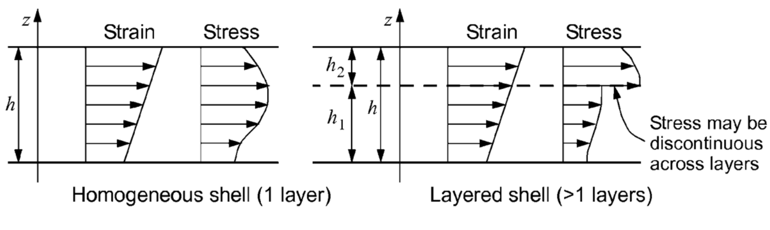
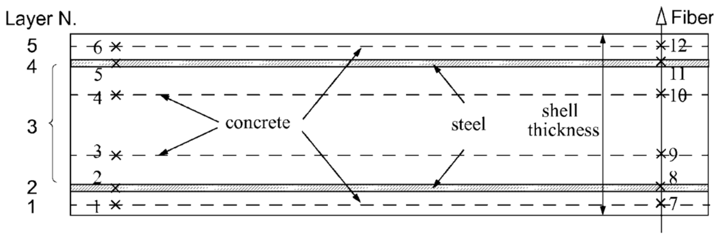
Exercise 3 – Impact on Cooling Tower (2)

- Shell model:



- Layers:



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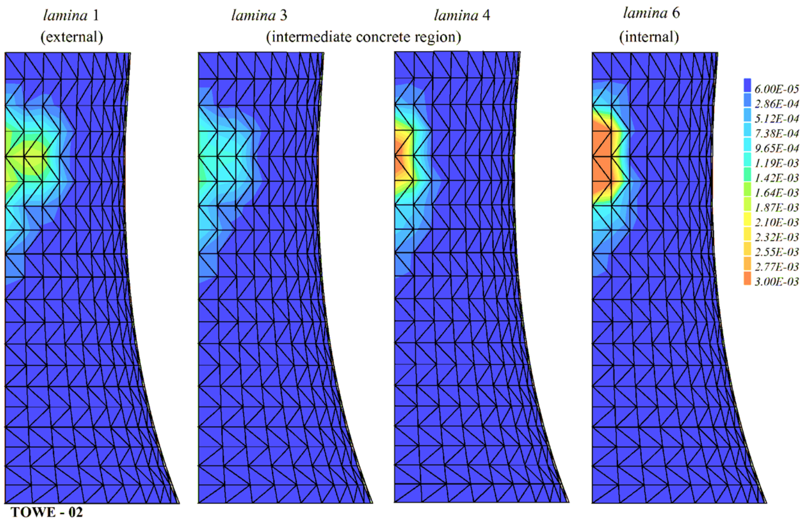
Exercise 3 – Impact on Cooling Tower (3)

- Results - equivalent plastic strain for the 4 concrete laminae at 100 ms:



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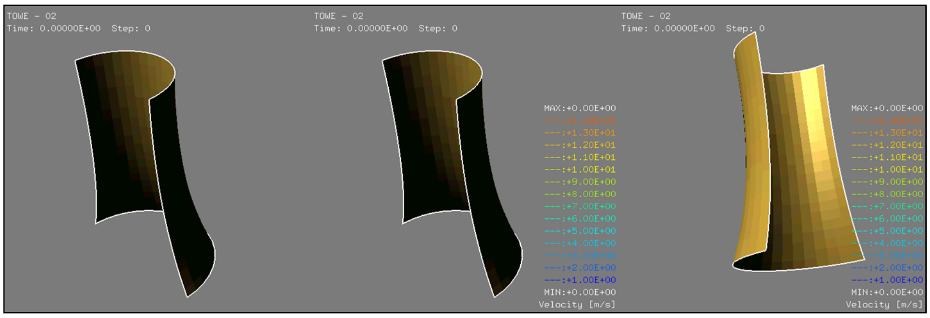
Exercise 3 – Impact on Cooling Tower (4)



Deformation *Velocities* *Velocities (2)*

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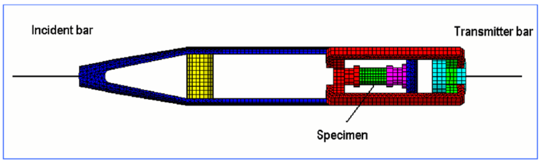
Exercise 4 – Hopkinson Bar

JRC Large Dynamic Test Facility (HopLab)



Fast Compression Test on Ductile Metallic Specimen

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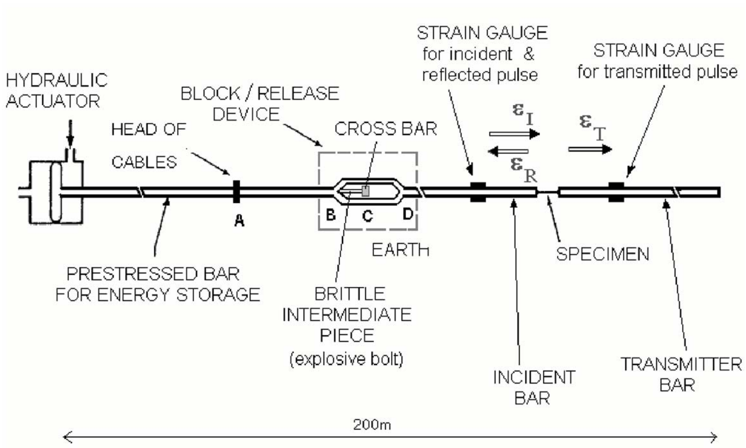
Exercise 4 – Hopkinson Bar (2)

Functioning Principle (Hopkinson):



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Exercise 4 – Hopkinson Bar (3)

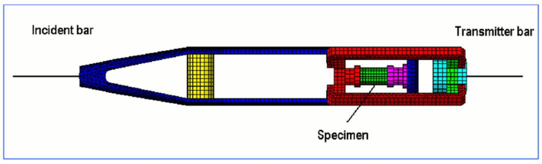
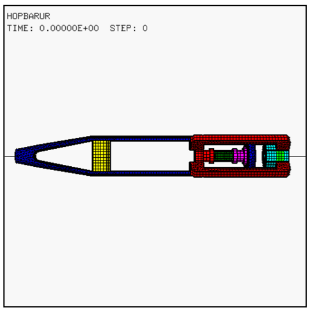
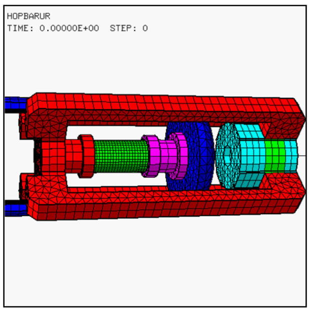


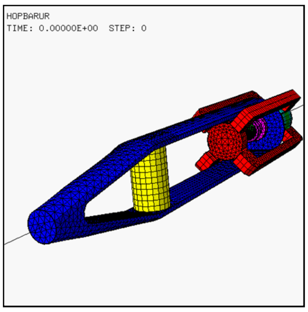
Figure 2: Incident and transmitter inversion cages

Exercise 4 – Hopkinson Bar (4)

Numerical simulation : geometrical model

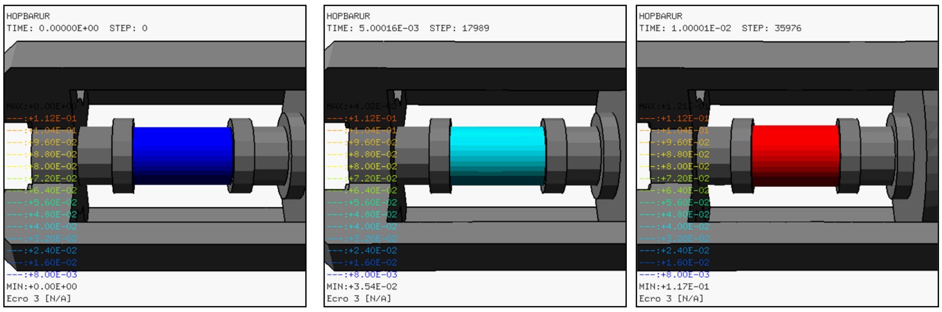






Exercise 4 – Hopkinson Bar (5)

Numerical simulation : specimen plastification



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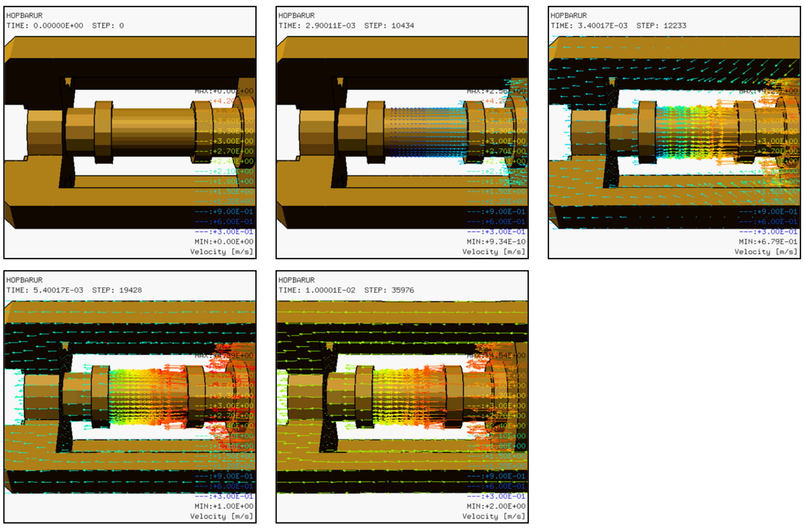
Exercise 4 – Hopkinson Bar (6)

Numerical simulation : velocities



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Exercise 4 – Hopkinson Bar (7)

Numerical simulation : results animation

EUROPLEXUS (C)

Hopkinson Bar

F. Casadei

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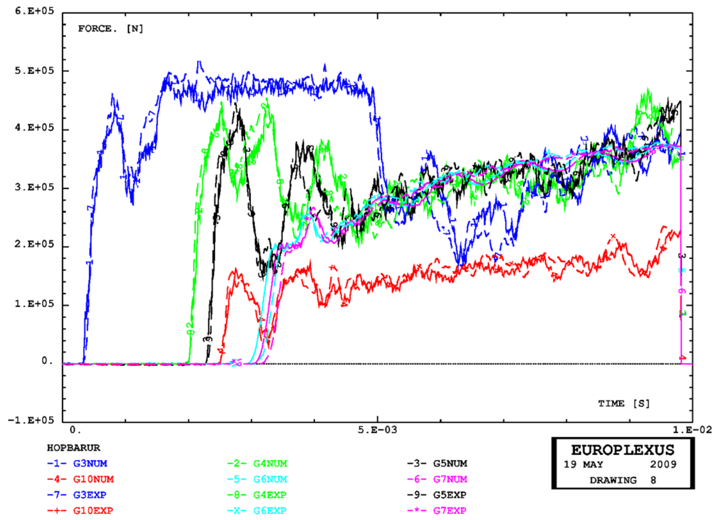
Exercise 4 – Hopkinson Bar (8)

Comparison experiment vs. calculation :



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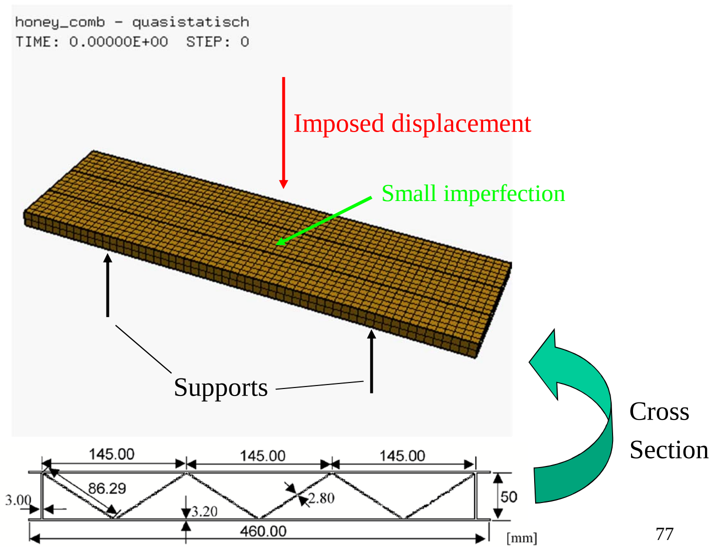
Exercise 5 – Honeycomb buckling



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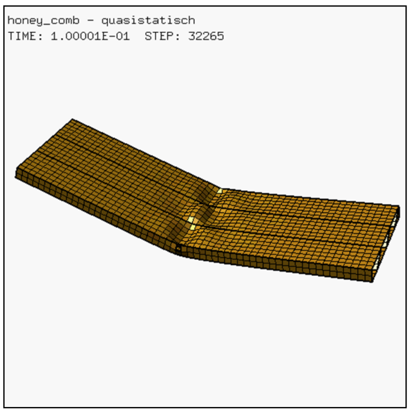
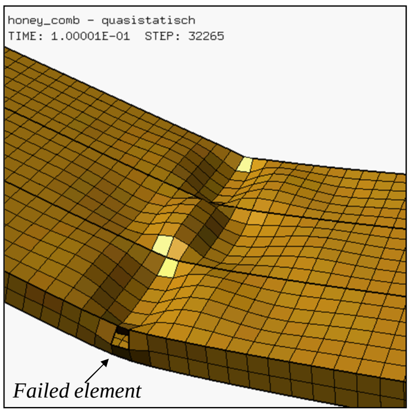
Exercise 5 – Honeycomb buckling (2)





Final deformed shape

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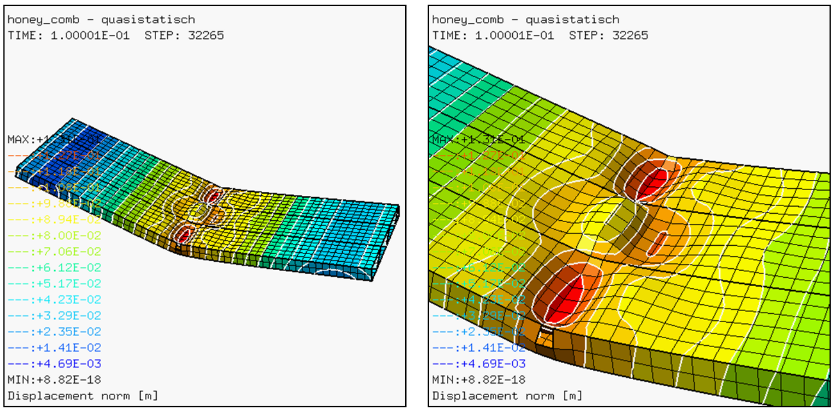
Exercise 5 – Honeycomb buckling (3)



Final displacement norm (buckling mode)

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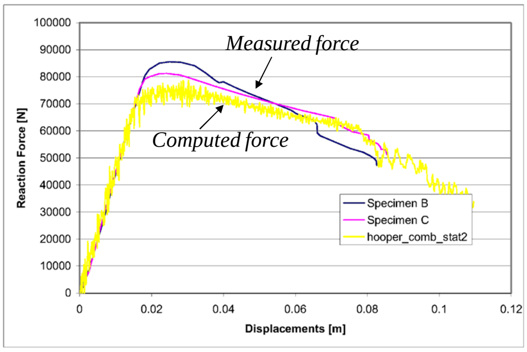
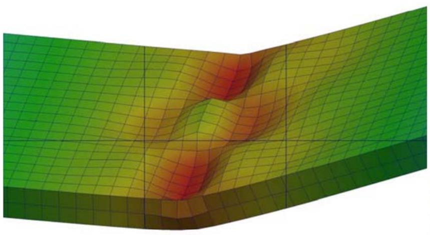
Exercise 5 – Honeycomb buckling (4)



Comparison experiment / calculation



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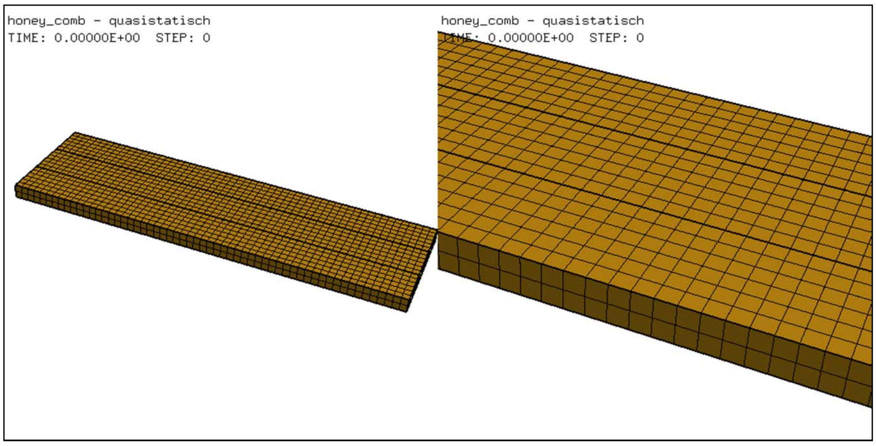
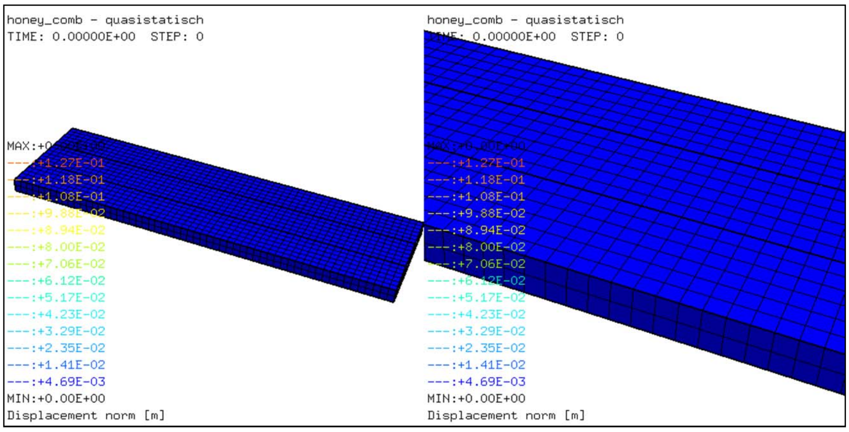
Exercise 5 – Honeycomb buckling (5)



Deformation

Exercise 5 – Honeycomb buckling (6)



Displacement